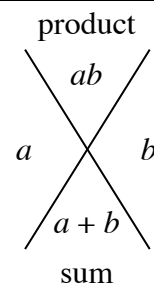


DIAMOND PROBLEMS

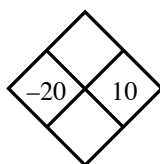
1.1.1

In every Diamond Problem, the product of the two side numbers (left and right) is the top number and their sum is the bottom number.

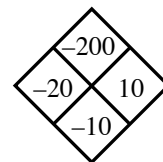
Diamond Problems are an excellent way of practicing addition, subtraction, multiplication, and division of positive and negative integers, decimals and fractions. They have the added benefit of preparing students for factoring binomials in algebra.



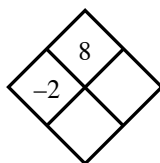
Example 1



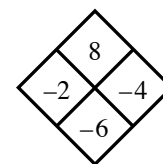
The top number is the product of -20 and 10 , or -200 . The bottom number is the sum of -20 and 10 , or $-20 + 10 = -10$.



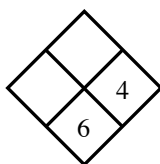
Example 2



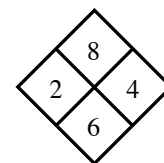
The product of the right number and -2 is 8 . Thus, if you divide 8 by -2 you get -4 , the right number. The sum of -2 and -4 is -6 , the bottom number.



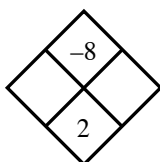
Example 3



To get the left number, subtract 4 from 6 , $6 - 4 = 2$. The product of 2 and 4 is 8 , the top number.



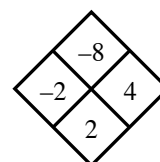
Example 4



The easiest way to find the side numbers in a situation like this one is to look at all the pairs of factors of -8 . They are:

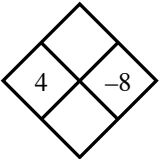
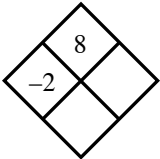
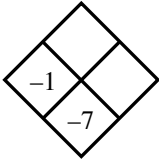
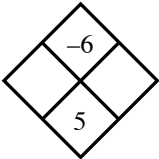
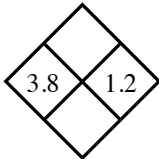
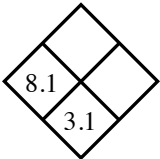
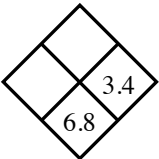
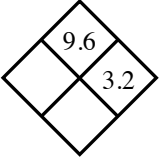
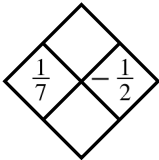
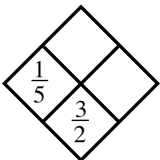
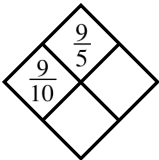
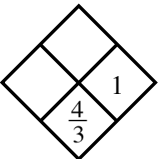
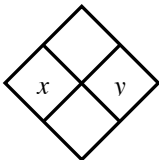
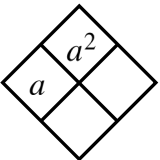
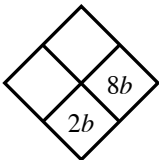
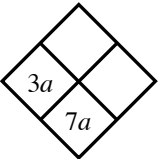
-1 and 8 , -2 and 4 , -4 and 2 , and -8 and 1 .

Only one of these pairs has a sum of 2 : -2 and 4 . Thus, the side numbers are -2 and 4 .



Problems

Complete each of the following Diamond Problems.

1. 
2. 
3. 
4. 
5. 
6. 
7. 
8. 
9. 
10. 
11. 
12. 
13. 
14. 
15. 
16. 

Answers

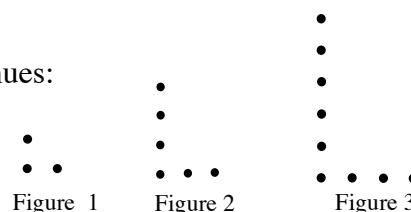
1. -32 and -4
2. -4 and -6
3. -6 and 6
4. 6 and -1
5. 4.56 and 5
6. -5 and -40.5
7. 3.4 and 11.56
8. 3 and 6.2
9. $-\frac{1}{14}$ and $-\frac{5}{14}$
10. $\frac{13}{10}$ and $\frac{13}{50}$
11. 2 and $\frac{29}{10}$
12. $\frac{1}{3}$ and $\frac{1}{3}$
13. xy and $x + y$
14. a and $2a$
15. $-6b$ and $-48b^2$
16. $4a$ and $12a^2$

Students are asked to use their observations and pattern recognition skills to extend patterns and predict the number of dots that will be in a figure that is too large to draw. Later, variables will be used to describe the patterns.

Example

Examine the dot pattern at right. Assuming the pattern continues:

- Draw Figure 4.
- How many dots will be in Figure 10?



Solution:

The horizontal dots are one more than the figure number and the vertical dots are even numbers (or, twice the figure number).

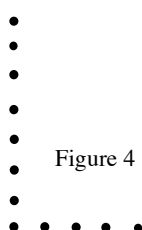
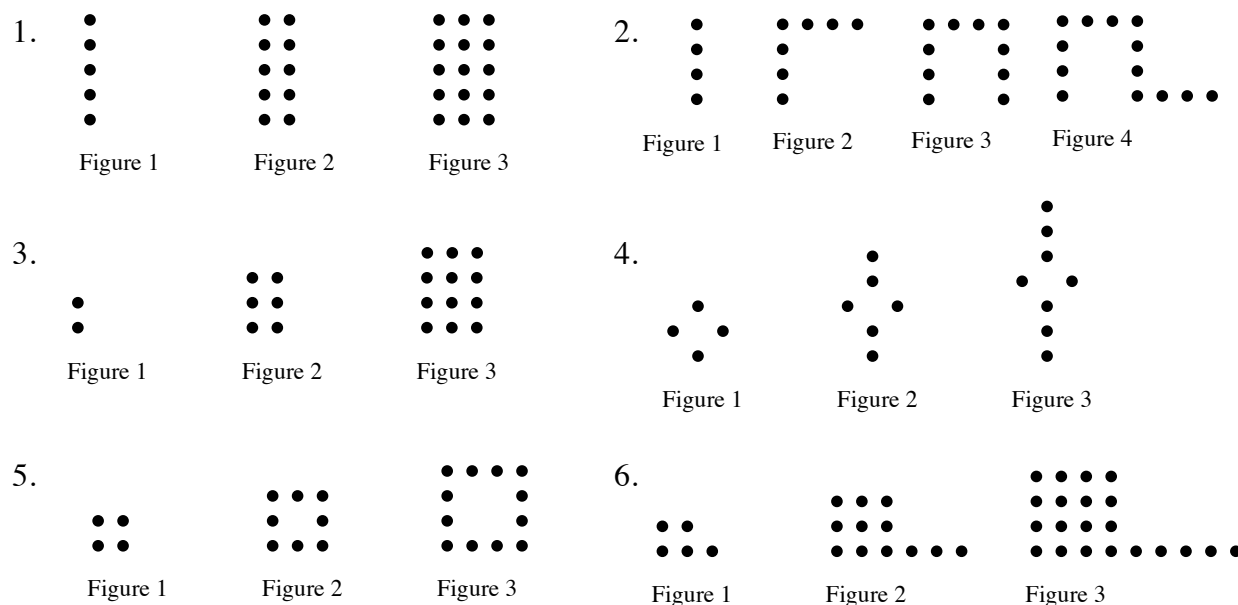


Figure 1 has 3 dots, Figure 2 has 6 dots, and Figure 3 has 9 dots. The number of dots is the figure number multiplied by three.

Figure 10 has 30 dots.

Problems

For each dot pattern, draw the next figure and determine the number of dots in Figure 10.



Answers

1. 50 dots

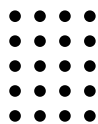


Figure 4

2. 31 dots

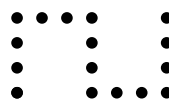


Figure 5

3. 110 dots

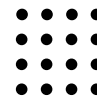


Figure 4

4. 22 dots

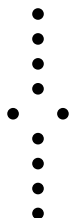


Figure 4

5. 40 dots

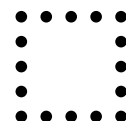


Figure 5

6. 140 dots

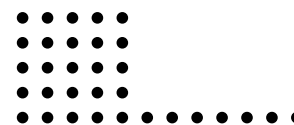


Figure 4

The graphing that was started in earlier grades is now extended to include negative values, and students will graph algebraic equations with two variables.

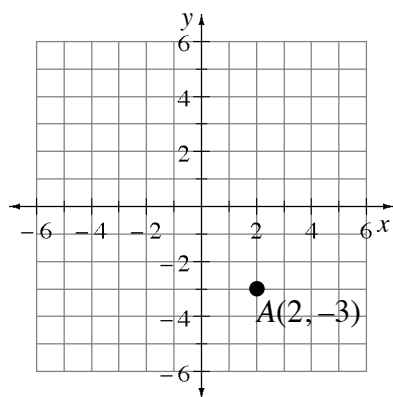
GRAPHING POINTS

Points on a coordinate graph are identified by two numbers in an ordered pair written as (x, y) . The first number is the x -coordinate of the point and the second number is the y -coordinate. Taken together, the two coordinates name exactly one point on the graph. The examples below show how to place a point on an xy -coordinate graph.

Example 1

Graph point $A(2, -3)$.

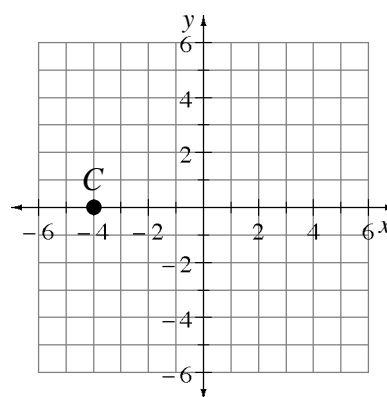
Go right 2 units from the origin $(0, 0)$, then go down 3 units. Mark the point.



Example 2

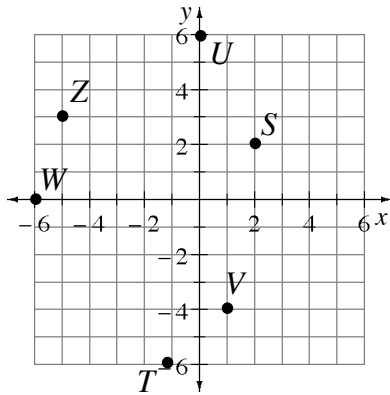
Plot the point $C(-4, 0)$ on a coordinate grid.

Go to the left from the origin 4 units, but do not go up or down. Mark the point.



Problems

1. Name the coordinate pair for each point shown on the grid below.
2. Use the ordered pair to locate each point on a coordinate grid. Place a “dot” at each point and label it with its letter name.



$K(0, -4)$

$L(-5, 0)$

$M(-2, -3)$

$N(-2, 3)$

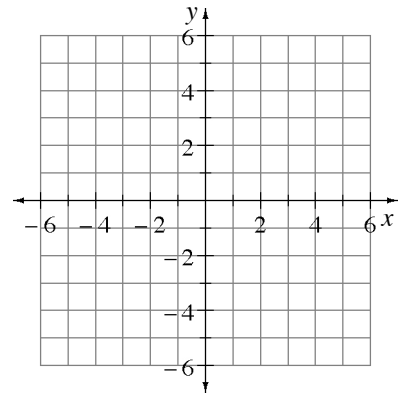
$O(2, -3)$

$P(-4, -6)$

$Q(4, -5)$

$R(-5, -4)$

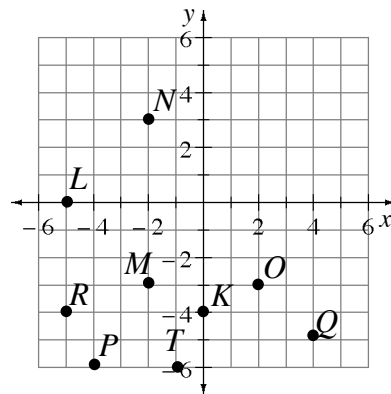
$T(-1, -6)$



Answers

1. $S(2, 2)$
 $T(-1, -6)$
 $U(0, 6)$
 $V(1, -4)$
 $W(-6, 0)$
 $Z(-5, 3)$

2.



WRITING EQUATIONS FOR WORD PROBLEMS (THE 5-D PROCESS)

1.1.3 Math Notes

At first students used the 5-D Process to solve problems. However, solving complicated problems with the 5-D Process can be time consuming and it may be difficult to find the correct solution if it is not an integer. The patterns developed in the 5-D Process can be generalized by using a variable to write an equation. Once you have an equation for the problem, it is often more efficient to solve the equation than to continue to use the 5-D Process. Most of the problems here will not be complex so that you can practice writing equations using the 5-D Process. The same example problems previously used are used here except they are now extended to writing and solving equations.

Example 1

A box of fruit has three times as many nectarines as grapefruit. Together there are 36 pieces of fruit. How many pieces of each type of fruit are there?

Describe: Number of nectarines is three times the number of grapefruit.
Number of nectarines plus number of grapefruit equals 36.

	Define		Do	Decide
	# of Grapefruit	# of Nectarines	Total Pieces of Fruit	36?
Trial 1:	11	33	44	too high
Trial 2:	10	30	40	too high

After several trials to establish a pattern in the problem, you can generalize it using a variable. Since we could try any number of grapefruit, use x to represent it. The pattern for the number of oranges is three times the number of grapefruit, or $3x$. The total pieces of fruit is the sum of column one and column two, so our table becomes:

	Define		Do	Decide
	# of Grapefruit	# of Nectarines	Total Pieces of Fruit	36?
	x	$3x$	$x + 3x$	$= 36$

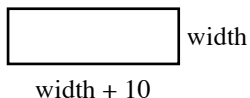
Since we want the total to agree with the check, our equation is $x + 3x = 36$. Simplifying this yields $4x = 36$, so $x = 9$ (grapefruit) and then $3x = 27$ (nectarines).

Declare: There are 9 grapefruit and 27 nectarines.

Example 2

The perimeter of a rectangle is 120 feet. If the length of the rectangle is 10 feet more than the width, what are the dimensions (length and width) of the rectangle?

Describe/Draw:



	Define		Do	Decide
	Width	Length	Perimeter	120?
Trial 1:	10	25	$(10 + 25) \cdot 2 = 70$	too low
Trial 2:	20	30	100	too low

Again, since we could guess any width, we labeled this column x . The pattern for the second column is that it is 10 more than the first: $x + 10$. The perimeter is found by multiplying the sum of the width and length by 2. Our table now becomes:

	Define		Do	Decide
	Width	Length	Perimeter	120?
	x	$x + 10$	$(x + x + 10) \cdot 2$	$= 120$

Solving the equation:

$$\begin{aligned} (x + x + 10) \cdot 2 &= 120 \\ 2x + 2x + 20 &= 120 \\ 4x + 20 &= 120 \\ 4x &= 100 \end{aligned}$$

So $x = 25$ (width) and $x + 10 = 35$ (length).

Declare: The width is 25 feet and the length is 35 feet.

Example 3

Jorge has some dimes and quarters. He has 10 more dimes than quarters and the collection of coins is worth \$2.40. How many dimes and quarters does Jorge have?

Describe: The number of quarters plus 10 equals the number of dimes.
The total value of the coins is \$2.40.

	Define				Do	Decide
	Quarters	Dimes	Value of Quarters	Value of Dimes	Total Value	\$2.40?
Trial 1:	10	20	2.50	2.00	4.50	too high
Trial 2:	8	18	2.00	1.80	3.80	too high
	x	$x + 10$	$0.25x$	$0.10(x + 10)$	$0.25x + 0.10(x + 10)$	

Since you need to know both the number of coins and their value, the equation is more complicated. The number of quarters becomes x , but then in the table the Value of Quarters column is $0.25x$. Thus the number of dimes is $x + 10$, but the value of dimes is $0.10(x + 10)$. Finally, to find the numbers, the equation becomes $0.25x + 0.10(x + 10) = 2.40$.

$$\begin{aligned}
 \text{Solving the equation: } & 0.25x + 0.10x + 1.00 = 2.40 \\
 & 0.35x + 1.00 = 2.40 \\
 & 0.35x = 1.40 \\
 & x = 4.00
 \end{aligned}$$

Declare: There are 4 quarters worth \$1.00 and 14 dimes worth \$1.40 for a total value of \$2.40.

Problems

Start the problems using the 5-D Process. Then write an equation. Solve the equation.

1. A wood board 100 centimeters long is cut into two pieces. One piece is 26 centimeters longer than the other. What are the lengths of the two pieces?
2. Thu is five years older than her brother Tuan. The sum of their ages is 51. What are their ages?
3. Tomás is thinking of a number. If he triples his number and subtracts 13, the result is 305. Of what number is Tomás thinking?
4. Two consecutive numbers have a sum of 123. What are the two numbers?
5. Two consecutive even numbers have a sum of 246. What are the numbers?
6. Joe's age is three times Aaron's age and Aaron is six years older than Christina. If the sum of their ages is 149, what is Christina's age? Joe's age? Aaron's age?

7. Farmer Fran has 38 barnyard animals, consisting of only chickens and goats. If these animals have 116 legs, how many of each type of animal are there?
8. A wood board 156 centimeters long is cut into three parts. The two longer parts are the same length and are 15 centimeters longer than the shortest part. How long are the three parts?
9. Juan has 15 coins, all nickels and dimes. This collection of coins is worth 90¢. How many nickels and dimes are there? (Hint: Create separate column titles for “Number of Nickels,” “Value of Nickels,” “Number of Dimes,” and “Value of Dimes.”)
10. Tickets to the school play are \$ 5.00 for adults and \$ 3.50 for students. If the total value of all the tickets sold was \$2517.50 and 100 more students bought tickets than adults, how many adults and students bought tickets?
11. A wood board 250 centimeters long is cut into five pieces: three short ones of equal length and two that are both 15 centimeters longer than the shorter ones. What are the lengths of the boards?
12. Conrad has a collection of three types of coins: nickels, dimes, and quarters. There is an equal amount of nickels and quarters but three times as many dimes. If the entire collection is worth \$9.60, how many nickels, dimes, and quarters are there?

Answers (Equations may vary.)

- | | |
|---|--|
| 1. $x + (x + 26) = 100$
The lengths of the boards are 37 cm and 63 cm. | 2. $x + (x + 5) = 51$
Thu is 28 years old and her brother is 23 years old. |
| 3. $3x - 13 = 305$
Tomás is thinking of the number 106. | 4. $x + (x + 1) = 123$
The two consecutive numbers are 61 and 62. |
| 5. $x + (x + 2) = 246$
The two consecutive even numbers are 122 and 124. | 6. $x + (x + 6) + 3(x + 6) = 149$
Christine is 25, Aaron is 31, and Joe is 93 years old. |
| 7. $2x + 4(38 - x) = 116$
Farmer Fran has 20 goats and 18 chickens. | 8. $x + (x + 15) + (x + 15) = 156$
The lengths of the boards are 42, 57, and 57 cm. |
| 9. $0.05x + 0.10(15 - x) = 0.90$
Juan has 12 nickels and 3 dimes. | 10. $\$5x + \$3.50(x + 100) = 2517.50$
There were 255 adult and 355 student tickets purchased for the play. |
| 11. $3x + 2(x + 15) = 250$
The lengths of the boards are 44 and 59 cm. | 12. $0.05x + 0.25x + 0.10(3x) = 9.60$
Conrad has 16 quarters, 16 nickels, and 48 dimes. |

BOX PLOTS

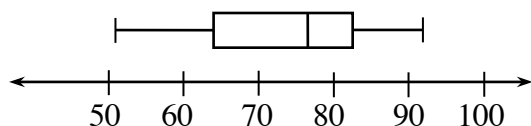
One way to display a distribution of one-variable numerical data is with a box plot. A box plot is the only display of data that clearly shows the median, quartiles, range, and outliers of a data set.

Example 1

Display this data in a box plot:

51, 55, 55, 62, 65, 72, 76, 78, 79, 82, 83, 85, 91, and 93.

- Since this data is already in order from least to greatest, the range is $93 - 51 = 42$. Thus you start with a number line with equal intervals from 50 to 100.
- The median of the set of data is 77. A vertical segment is drawn at this value above the number line.
- The median of the lower half of the data (the first quartile) is 62. A vertical segment is drawn at this value above the number line.
- The median of the upper half of the data (the third quartile) is 83. A vertical segment is drawn at this value above the number line.
- A box is drawn between the first and third quartiles.
- Place a vertical segment at the minimum value (51) and at the maximum value (93). Use a line segment to connect the minimum to the box and the maximum to the box.

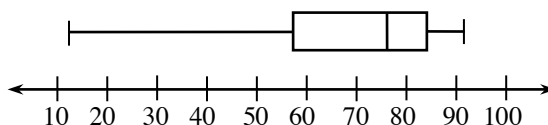


Example 2

Display this data in a box plot:

62, 65, 93, 51, 12, 79, 85, 55, 72, 78, 83, 91, and 76.

- Place the data in order from least to greatest: 12, 51, 55, 62, 65, 72, 76, 78, 79, 83, 85, 91, 93. The range is $93 - 12 = 81$. Thus you want a number line with equal intervals from 10 to 100.
- Find the median of the set of data: 76. Draw the line segment.
- Find the first quartile: $55 + 62 = 117$; $117 \div 2 = 58.5$. Draw the line segment.
- Find the third quartile: $83 + 85 = 168$; $168 \div 2 = 84$. Draw the line segment.
- Draw the box connecting the first and third quartiles. Place a line segment at the minimum value (12) and a line segment at the maximum value (93). Connect the minimum and maximum values to the box.

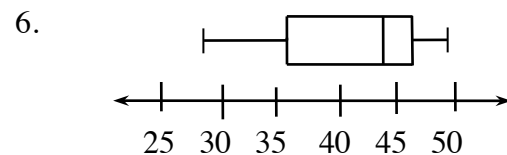
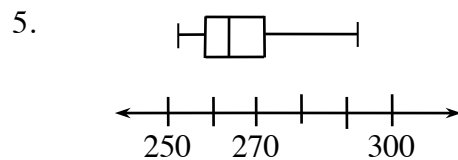
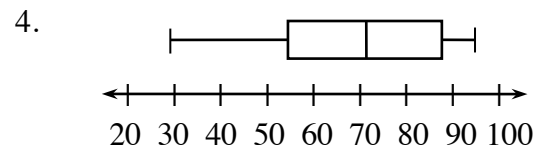
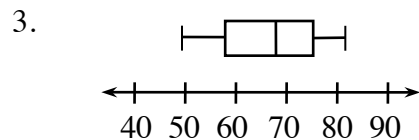
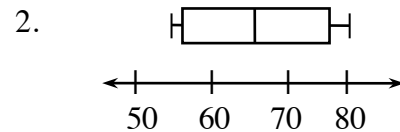
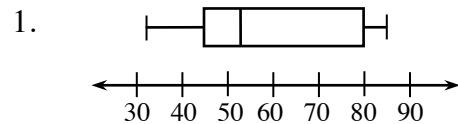


Problems

Create a box plot for each set of data.

- 45, 47, 52, 85, 46, 32, 83, 80, and 75.
- 75, 62, 56, 80, 72, 55, 54, and 80.
- 49, 54, 52, 58, 61, 72, 73, 78, 73, 82, 83, 73, 61, 67, and 68.
- 65, 35, 48, 29, 57, 87, 94, 68, 86, 73, 58, 74, 85, 91, 88, and 97.
- 265, 263, 269, 259, 267, 264, 253, 275, 264, 260, 273, 257, and 291.
- 48, 42, 37, 29, 49, 46, 38, 28, 45, 45, 35, 46.25, 34, 46, 46.5, 43, 46.5, 48, 41.25, 29, and 47.75.

Answers



A **proportion** is an equation stating the two ratios (fractions) are equal. Two values are in a proportional relationship if a proportion may be set up to relate the values.

For more information, see the Math Notes boxes in Lessons 1.2.2 and 7.2.5 of the *Core Connections, Course 3* text. For additional examples and practice, see the *Core Connections, Course 3* Checkpoint 3 materials.

Example 1

The average cost of a pair of designer jeans has increased \$15 in 4 years. What is the unit growth rate (dollars per year)?

Solution: The growth rate is $\frac{15 \text{ dollars}}{4 \text{ years}}$. To create a unit rate we need a denominator of “one.”

$\frac{15 \text{ dollars}}{4 \text{ years}} = \frac{x \text{ dollars}}{1 \text{ year}}$. Using a Giant One: $\frac{15 \text{ dollars}}{4 \text{ years}} = \frac{4}{4} \cdot \frac{x \text{ dollars}}{1 \text{ year}} \Rightarrow 3.75 \frac{\text{dollars}}{\text{year}}$.

Example 2

Ryan’s famous chili recipe uses 3 tablespoons of chili powder for 5 servings. How many tablespoons are needed for the family reunion needing 40 servings?

Solution: The rate is $\frac{3 \text{ tablespoons}}{5 \text{ servings}}$ so the problem may be written as a proportion: $\frac{3}{5} = \frac{t}{40}$.

One method of solving the proportion is to use the Giant One:

$$\frac{3}{5} = \frac{t}{40} \Rightarrow \frac{3}{5} \cdot \frac{8}{8} = \frac{24}{40} \Rightarrow t = 24$$

Another method is to use cross multiplication:

$$\frac{3}{5} = \frac{t}{40}$$

$$\frac{3}{5} \times \frac{t}{40}$$

$$5 \cdot t = 3 \cdot 40$$

$$5t = 120$$

$$t = 24$$

Finally, since the unit rate is $\frac{3}{5}$ tablespoon per serving, the equation $t = \frac{3}{5}s$ represents the general proportional situation and one could substitute the number of servings needed into the equation: $t = \frac{3}{5} \cdot 40 = 24$. Using any method the answer is 24 tablespoons.

Example 3

Based on the table at right, what is the unit growth rate (meters per year)?

Solution: $\frac{2 \text{ meters}}{10 \text{ years}} = \frac{2 \text{ meters} \cdot \frac{1}{10}}{10 \text{ years} \cdot \frac{1}{10}} = \frac{\frac{1}{5} \text{ meter}}{1 \text{ year}} = \frac{1}{5} \frac{\text{meters}}{\text{year}}$

height (m)	15	17
time (years)	75	85

$\xrightarrow{+2}$
 $\xrightarrow{+10}$

Problems

For problems 1 through 10 find the unit rate. For problems 11 through 25, solve each problem.

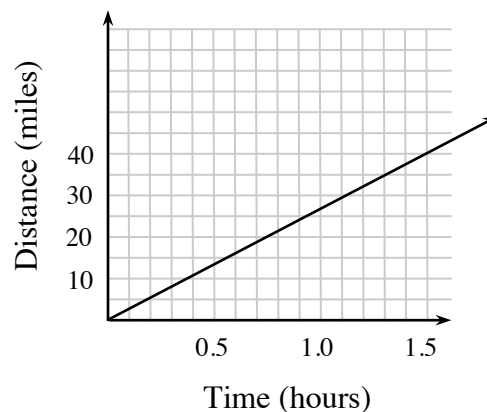
- Typing 731 words in 17 minutes (words per minute)
- Reading 258 pages in 86 minutes (pages per minute)
- Buying 15 boxes of cereal for \$43.35 (cost per box)
- Scoring 98 points in a 40 minute game (points per minute)
- Buying $2\frac{1}{4}$ pounds of bananas cost \$1.89 (cost per pound)
- Buying $\frac{2}{3}$ pounds of peanuts for \$2.25 (cost per pound)
- Mowing $1\frac{1}{2}$ acres of lawn in $\frac{3}{4}$ of a hour (acres per hour)
- Paying \$3.89 for 1.7 pounds of chicken (cost per pound)

9.

weight (g)	6	8	12	20
length (cm)	15	20	30	50

What is the weight per cm?

- For the graph at right, what is the rate in miles per hour?
- If a box of 100 pencils costs \$4.75, what should you expect to pay for 225 pencils?
- When Amber does her math homework, she finishes 10 problems every 7 minutes. How long will it take for her to complete 35 problems?
- Ben and his friends are having a TV marathon, and after 4 hours they have watched 5 episodes of the show. About how long will it take to complete the season, which has 24 episodes?
- The tax on a \$600 vase is \$54. What should be the tax on a \$1700 vase?

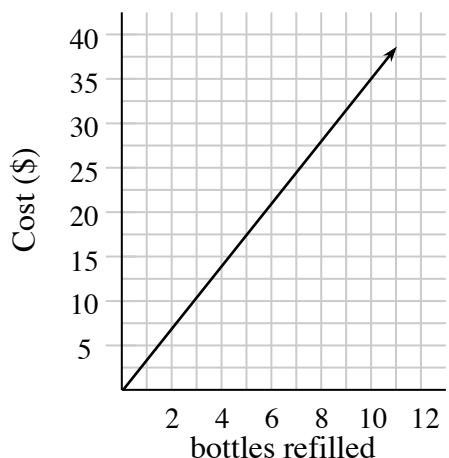


15. Use the table at right to determine how long it will take the Spirit club to wax 60 cars.

cars waxed	8	16	32
hours	3	6	12

16. While baking, Evan discovered a recipe that required $\frac{1}{2}$ cups of walnuts for every $2\frac{1}{4}$ cups of flour. How many cups of walnuts will he need for 4 cups of flour?

17. Based on the graph, what would the cost to refill 50 bottles?



18. Sam grew $1\frac{3}{4}$ inches in $4\frac{1}{2}$ months. How much should he grow in one year?

19. On his afternoon jog, Chris took 42 minutes to run $3\frac{3}{4}$ miles. How many miles can he run in 60 minutes?

20. If Caitlin needs $1\frac{1}{3}$ cans of paint for each room in her house, how many cans of paint will she need to paint the 7-room house?

21. Stephen receives 20 minutes of video game time every 45 minutes of dog walking he does. If he wants 90 minutes of game time, how many hours will he need to work?

22. Sarah's grape vine grew 15 inches in 6 weeks, write an equation to represent its growth after t weeks.

23. On average Max makes 45 out of 60 shots with the basketball, write an equation to represent the average number of shots made out of x attempts.

24. Write an equation to represent the situation in problem 14.

25. Write an equation to represent the situation in problem 17.

Answers

- | | | | |
|---|---|---|---|
| 1. $43 \frac{\text{words}}{\text{minute}}$ | 2. $3 \frac{\text{pages}}{\text{minute}}$ | 3. $2.89 \frac{\$}{\text{box}}$ | 4. $2.45 \frac{\text{points}}{\text{minute}}$ |
| 5. $0.84 \frac{\$}{\text{pound}}$ | 6. $3.38 \frac{\$}{\text{pound}}$ | 7. $2 \frac{\text{acres}}{\text{hour}}$ | 8. $2.29 \frac{\$}{\text{pound}}$ |
| 9. $\frac{2}{5} \frac{\text{grams}}{\text{centimeter}}$ | 10. $\approx 27 \frac{\text{miles}}{\text{hour}}$ | 11. \$10.69 | 12. 24.5 min. |
| 13. 19.2 hours | 14. \$153 | 15. 22.5 hours | 16. $\frac{8}{9}$ cup |
| 17. \$175 | 18. $4\frac{2}{3}$ inches | 19. ≈ 5.36 miles | 20. $9\frac{1}{3}$ cans |
| 21. $3\frac{3}{8}$ hours | 22. $g = \frac{5}{2}t$ | 23. $s = \frac{3}{4}x$ | 24. $t = 0.09c$ |
| 25. $C = 3.5b$ | | | |